**Eigenfunctions/values of**

So that was fun, but laborious. There is an algebraic way of determining the eigenvectors using raising/lowering operators. So let’s do that…We’ll illustrate with the ℓ = 1, s = ½ case again.

**Algebraic approach to obtaining eigenvectors**

The method mentioned is to use the raising and lowering operators J± to obtain the eigenvectors. Let’s illustrate it using the case just covered: ℓ = 1, and s = ½. Then according to the rules above, j can range between jmin = |ℓ-s| = ½ and jmax = ℓ+s = 3/2. And mj = -1/2, ½ in the first case, and mj = -3/2, -1/2, ½, 3/2 in the second. Those are the eigenvalues. Now let’s determine the eigenvectors. The procedure starts with the case where the orbital and spin angular momentum states are ‘aligned’, i.e. with j = jmax and mj = jmax.

**Case j = 3/2, mj = 3/2**

According to the equation we must have:



But the only possible mℓ and ms values that can add up to 3/2 are when mℓ = 1, and ms = ½, their maximum values. And so there is only one uncoupled state in the sum. So we have:



That was easy!

**Case j = 3/2, mj = 1/2**

We can get this state by applying the lowering operator, J- to the state above. We have:



upon normalization.

**Case: j = 3/2, mj = -1/2**

We can apply the lowering operator again to the state above. But it is actually easier to apply the raising operator to state below – so skip to that one first and then come back.



upon normalization.

**Case j = 3/2, mj = -3/2**

According to the equation we must have:



But the only possible mℓ and ms values that can add up to 3/2 are when mℓ = -1, and ms = -½, their minimum values. And so there is only one uncoupled state in the sum. So we have:



**Case j = ½, mj = 1/2**

According to the equation we have:



because there are two sets of mℓ, ms values that add to ½. To determine these coefficients, we simply use the fact that J+ operating on this ket must be 0. And so we must have:



So we have:



There is a minus sign difference from the expression above – but that doesn’t matter as it is still normalized. The minus sign has no physical significance.

**Case j = ½, mj = -1/2**

And we can get this one by applying the lowering operator to the j = ½, mj = ½ eigenket above. But since you’re perhaps now familiar with that procedure, let’s consider an alternate method. Use the equation to get the general form:



And note that the eigenket |j=3/2,mj=-1/2> contains the same uncoupled kets,



Well, our present ket |j=1/2,mj=-1/2> must be orthogonal to this one and so we can determine the coefficients by demanding their inner product be 0. We would have:



And so,



upon normalization. Again the minus sign difference is irrelevant. We could have done similarly with the j=1/2, mj = ½ ket as well. We’ll note these are the results we got before.